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CORRELATION EQUATIONS FOR FREE CONVECTION HEAT TRANSFER IN HORIZONTAL LAYERS OF AIR AND WATER

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Abstract—New experimental measurements are reported on the natural convective heat transport through a horizontal layer of air, covering the Rayleigh number range from sub-critical to 4×10^6. When these data are combined with Goldstein and Chu's data for air, the full set of data points are demonstrated to bear a Nusselt number dependence which is asymptotic to a 1/3 power on the Rayleigh number as the Rayleigh number approaches infinity. The asymptotic coefficient of proportionality is consistent with that predicted by a simple "conduction layer model" which is described. Knowledge of the asymptote has permitted a simple but accurate correlation equation to be obtained, valid for the full range of Rayleigh number. By extension, a similar correlation equation is also obtained for water.

NOMENCLATURE

C_t, constant in equation (2) [dimensionless];
d, depth of fluid layer [m];
g, acceleration of gravity [m/s^2];
h, convective heat-transfer coefficient, across fluid layer [W/m^{2}K];
k, thermal conductivity of fluid [W/m K];
Nu, Nusselt number, $= hd/k$ [dimensionless];
Pr, Prandtl number, $= \nu/\alpha$;
Ra, Rayleigh number, $= \dfrac{g\beta(T_1 - T_2)d^3}{\nu\alpha}$ [dimensionless];
T_1, T_2, temperature of lower and upper plate respectively [K];
ΔT_{BL}, temperature difference across boundary layer [K].

Greek symbols

α, thermal diffusivity of fluid [m^2/s];
β, thermal expansion coefficient of fluid [K^{-1}];
Δ_t, conduction thickness of boundary layer [m];
ν, kinematic viscosity of fluid [m^2/s];
ϕ, angle of tilt of surface from horizontal [deg].

INTRODUCTION

THE CONVECTIVE motion and heat transfer occurring in a horizontal layer of fluid heated from below has been the subject of a large number of investigations in recent years, the subject having been recently reviewed by Plate [1]. For Rayleigh numbers less than a critical value of 1708 it is well known that the fluid layer is stagnant and the Nusselt number is unity. At Rayleigh numbers slightly greater than critical, the fluid flow consists of steady rolls. Subsequent increases in the Rayleigh number produces flows of increasing complexity, the exact nature of the flow pattern depending on the Prandtl number. Thus the transition to time-unsteady flow varies from $Ra \simeq 5500$ for $Pr = 0{\cdot}7$ to $Ra \simeq 55\,000$ for $Pr = 8500$ [2]. For Rayleigh number greater than about 10^6 the structure of the flow is generally considered to be fully turbulent.

Measurements of the fluid temperature profile at high Rayleigh number shows a boundary-layer type structure, there being a nearly isothermal inner core with high temperature gradients close to the boundary surfaces. A surprising observation resulting from the measurements on water is a reversal in the temperature gradient in the inner core [3]. In observing the flow structure at high Rayleigh number with water, Goldstein and Chu [3] noted the existence of "thermals" (buoyancy-driven masses of fluid moving away from the boundary surfaces); and "stable blobs" (coalesced thermals which have reached the opposite boundary). It appears that the "thermal blobs" are the explanation of the reversed temperature gradient. Neither of these phenomena was observed when those authors performed similar studies on air [4].

Correlation equations for the heat transfer across the layer covering the full Rayleigh and Prandtl number range were last given in 1960 by O'Toole and Silveston [5]. Using all data available at that time these authors divided the flow into three Rayleigh number regions: initial ($1700 < Ra < 3500$); laminar ($3500 < Ra < 10^5$) and turbulent ($10^5 < Ra < 10^9$), and gave correlation equations for each range. Only in the turbulent region was the Nusselt number found to be Prandtl-number dependent. Consequently their equations for Nu have a discontinuity at $Ra = 10^5$. Since their study, a number of measurements of heat transfer have been reported, principally by Rossby [6], Goldstein and Chu [3], [4], Garon and Goldstein [7], Willis and Deardorff [8] and Krishnamurti [9], [2]. (Some of these studies, particularly the last two, were more concerned with the

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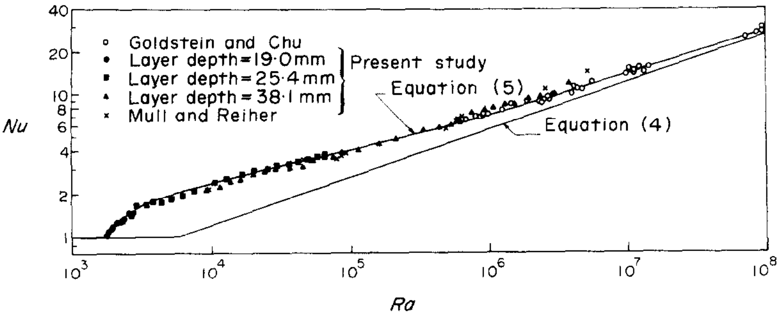

FIG. 1. Experimental data and correlation equations for air.

transition in the heat-transfer curves, than in the absolute value of the Nusselt number.) There have also been considerable developments in theory, some of which are predictive of the heat-transfer rates. In particular, the equation of Malkus and Veronis [10]:

$$Nu = 1+1{\cdot}44\left(1-\frac{1708}{Ra}\right) \tag{1}$$

successfully predicts the heat transfer in the steady roll region near the point of instability [11] (i.e. up to about $Ra \simeq 5500$); and the "independent mode" theory of Malkus and Veronis [10] as refined by Catton [12], appears to fit the data for liquids up to Rayleigh numbers of about 10^5.

The asymptotic behaviour of the Nusselt number as the Rayleigh number approaches infinity remains an unresolved problem. In an important paper, Malkus [13] in 1954, treated this asymptotic behaviour by maximizing the heat-transfer subject to two assumed constraints, and obtained: $Nu \propto Ra^{1/3}$. Later Herring [14], [15], using a numerical solution of the governing equations, but neglecting the non-linear interaction terms, also obtained the result $Nu \propto Ra^{1/3}$.

Experimental studies however have not always borne out the $Ra^{1/3}$ dependence at high Rayleigh number. Although Globe and Dropkin [16] found a 1/3 power dependence, O'Toole and Silveston [5] found $Nu \propto Ra^{0{\cdot}305}$ in their turbulent region. Rossby [6] reported different power laws for different Prandtl numbers, the index on the Rayleigh number ranging from 0·281 for silicone to 0·257 for mercury ($Pr \simeq 0{\cdot}025$). Recently Goldstein and Chu [3, 4] on the basis of careful experiments and a large number of data points in the high Rayleigh number range (10^6 to 10^8) obtained: $Nu \propto Ra^{0{\cdot}294}$ for air and $Nu \propto Ra^{0{\cdot}278}$ for water. On the basis of observing nearly twice the standard deviation of their data if they assumed a 1/3 power on the Rayleigh number, they concluded that "the 1/3 power law predicted by many theoretical analyses does not apply to the present experiments."

The present article presents some new experimental measurements of the heat transfer with air as the fluid, covering the Rayleigh number range from sub-critical to 4×10^6. When these data are combined with Goldstein and Chu's data for air (which lie in the range 5×10^5 to 10^8), the resultant total data set is shown to be asymptotic to a 1/3 power law, the coefficient being consistent with that predicted by a "conduction layer model", which is described. This fact permits simple correlation equations to be obtained, expressing accurately the Nusselt number dependence over the full Rayleigh number range. By extension, a similar but slightly more complex correlation for water is also obtained.

EXPERIMENT

The apparatus used in this experiment has been used previously to measure the critical Rayleigh number for tilted air layers [17] and to determine critical Rayleigh numbers and heat transfer in horizontal layers of air constrained by a honeycomb structure [18]. It has therefore been fully described elsewhere. It consists essentially of two parallel copper plates each 56 cm × 61 cm × 1 cm, the upper one cooled, the lower one heated by two separate circulating water streams so as to maintain a uniform temperature difference of the order of 10 K° between the plates. The heat flow leaving the lower plate is determined in the central 13 by 13 cm area by means of a nulling system involving a heat flux meter and a separate electrically heated plate. The plates are inserted in a vacuum (or pressure) vessel in which the pressure can be varied from 10 Pa to 700 kPa. The provision of varying the air pressure permits varying the Rayleigh number over a wide range without altering either the plate spacing or the temperature difference between the plates.

Runs were made at nominal plate spacings of 1·0 cm, 2·5 cm and 3·8 cm and the results in de-dimensionalized form are shown plotted in Fig. 1. Generally, if there is an overlap between data of different plate spacings, that of the lower spacing is to be taken as the more accurate, since in this instance the control area where the heat transfer is measured is more representative of the total flow structure.

Measurements of the heat transfer through air layers have also been made by Mull and Reiher [19], de Graaf and Van der Held [20], Willis and Deardorff [8] and Goldstein and Chu [4]. Mull and Reiher's data points are plotted in Fig. 1 and are seen to lie very close to the present data. De Graaf and Van der Held are in general agreement with the present study but show considerably more scatter. The data of Goldstein and

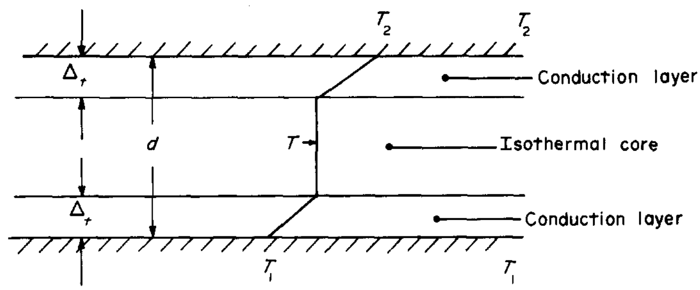

FIG. 2. Sketch illustrating conduction layer model.

Chu, all falling in the high Rayleigh number range, are also plotted in Fig. 1. They complement and extend the present data.

Willis and Deardorff's experiment was less aimed at the absolute magnitude of the heat transfer than in the location of the transition points in the heat-transfer curve which were first pointed out by Malkus. The present results also show these transition points and this can be seen more easily if one follows the results for the 3·8 cm spacing alone. The location of these transition points, determined by replotting the data on linear scale in the form $NuRa$ vs Ra and fitting a set of straight lines to the results, are consistent with the results of Willis and Deardorff.

CONDUCTION LAYER MODEL

Two of the authors (Hollands and Raithby [21]) have had some success in modelling natural convection phenomena by surrounding all rigid boundaries by a layer of stagnant fluid (called the conduction layer) and considering the heat transfer to be conducted through the region. The thickness of the conduction layer is such as to offer the same *local* resistance by conduction, as the actual boundary layer. In the case of a turbulent boundary layer, the thickness is determined completely locally. It depends only on the local angle of tilt of the surface, the local temperature difference across the boundary layer, and the fluid properties. On the basis of dimensional analysis the conduction layer thickness can be expressed as:

$$\Delta_t = \frac{1}{C_t}\left(\frac{\nu\alpha}{\beta g \Delta T_{BL}}\right)^{1/3} \cdot \frac{1}{A(\phi)} \tag{2}$$

where C_t is a (weak) function of the Prandtl number. $A(\phi)$ denotes a dependence on the local angle of tilt of the bounding surface from the horizontal. In the present problem $\phi = 0$; $A(0)$ is defined as unity.

The value of C_t can be determined by experiments conducted on a large single horizontal plate in an extensive fluid environment. In this instance equation (1) would predict $Nu_L = C_t Ra_L^{1/3}$. (The length dimension, L, representing the plate width, cancels out in this instance.) For this problem Fishenden and Saunders [22] recommend $Nu = 0{\cdot}14 Ra^{1/3}$ for air. Hassen and Mohammed [23] recently confirmed this value to within 4 per cent. Thus for air ($Pr \simeq 0{\cdot}7$), $C_t \simeq 0{\cdot}14$, and we shall use this value subsequently. Working with water ($Pr \simeq 7$), Fujii and Imura [24] report a value of $C_t = 0{\cdot}13$. To the authors' knowledge, this type of experiment has not been performed for higher Prandtl number fluids, so that values of C_t for these fluids are not available.

Figure 2 shows a sketch of the conduction layer model as applied to the present problem. The inner core of fluid is assumed to be perfectly mixed with eddy diffusion. The anticipated temperature profile (except for the small gradient reversal) is approximately consistent with that observed, as mentioned in the Introduction. Since the temperature drop across each conduction layer is one-half of the overall temperature difference [i.e. $\Delta T_{BL} = \frac{1}{2}(T_1 - T_2)$] the conduction-layer thickness on each surface is:

$$\Delta_t = \frac{1}{C_t}\left(\frac{2\nu\alpha}{g\beta(T_1 - T_2)}\right)^{1/3}.$$

The resistance to heat transfer lies in two layers of stagnant fluid, both of thickness Δ_t, so that the overall heat-transfer coefficient across the layer, h is given by:

$$h = \frac{k}{2\Delta_t} = \frac{C_t}{2^{4/3}} \cdot k\left(\frac{g\beta(T_1 - T_2)}{\nu\alpha}\right)^{1/3}.$$

Expressed dimensionally, this heat transfer is:

$$Nu = \frac{C_t}{2^{4/3}} Ra^{1/3} = 0{\cdot}0555 Ra^{1/3} \tag{3}$$

where C_t is taken as 0·14. Clearly it is impossible for the Nusselt number to be less than unity so that equation (3) cannot apply for $Ra < 2^4/C_t^3 = 5830$. This condition applies to the point where the two conduction layers just touch. For spacings less than this, heat is conducted directly across the fluid and $Nu = 1$. The assumption that, if two conduction layers overlap, the intermediate fluid can be taken as stagnant, is implicit in the conduction layer model [21]. Thus the conduction layer model would predict for the present situation

$$Nu = 1 \qquad Ra < 5830 \tag{4a}$$

$$Nu = 0{\cdot}0555 Ra^{1/3} \qquad Ra > 5830. \tag{4b}$$

While significant departures from equation (4) may be expected at intermediate values of Ra, where it would be anticipated that the relatively close proximity

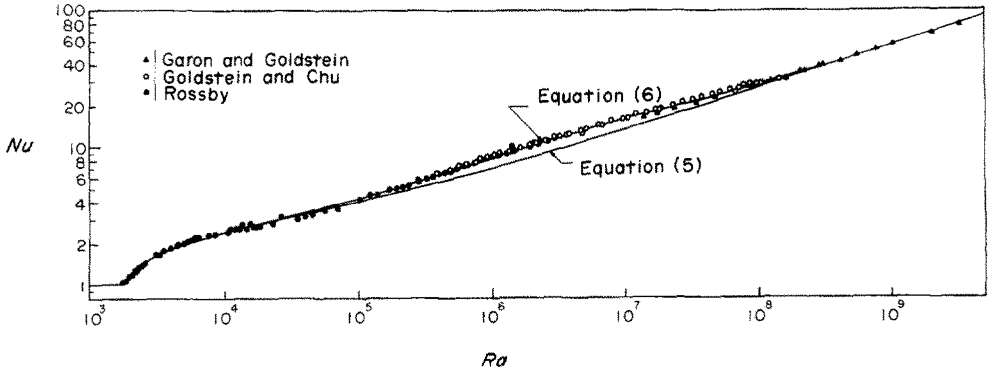

FIG. 3. Comparison of correlation equations with recent experimental data for water.

of the two boundary layers causes an interaction between them, the model would be expected to apply in the limit of very large Rayleigh number, i.e. large spacing, since in this instance the boundary layers should become essentially independent of each other. (In the case of laminar boundary layers on concentric cylinders and spheres, it has been found [21] that there is no measurable variation of the conduction layer thickness with spacing even down to (but not past) the point where the conduction layers meet.) Since it is known that for $Ra < 1708$, the fluid is stagnant and $Nu = 1$, equation (4) will also apply exactly for $Ra < 1708$.

Equation (4) is plotted in Fig. 1 to permit comparison with experiment. As expected, significant departures are observed for intermediate Rayleigh numbers. However, the data appear to be definitely asymptotic in equation 4(b). It would appear then on both theoretical and experimental grounds, that equation 4(b) represents the sought asymptotic behaviour of the Nusselt number as $Ra \to \infty$, at least for air.

CORRELATION EQUATIONS

Air

The manner in which the data approach 4(b) is of interest. Inspection of the plot shows that the difference in the Nusselt number between the data points and equation (4) remains relatively constant at approximately 1·5 except for Rayleigh numbers near critical; at the critical Rayleigh number the difference is zero. The formula for the behaviour of the convective Nusselt number in the vicinity of the critical condition, given by equation (1), behaves in a very similar manner to this difference in Nusselt numbers and hence it is useful to represent the Nusselt number by the sum of that given by (4) and (1); i.e. by:

$$Nu = 1 + 1{\cdot}44\left[1 - \frac{1708}{Ra}\right]^{\bullet} + \left[\left(\frac{Ra}{5830}\right)^{1/3} - 1\right]^{\bullet} \tag{5}$$

where a bracket: $[\ ]^{\bullet}$ indicates that if the argument inside the bracket is negative, the quantity is to be taken as zero. Equation (5) is plotted in Fig. 1. It is seen to fit the data very closely. It is important to note that the single equation (5) correlates the Nusselt number behaviour for the full range of Rayleigh number, i.e. from 0 to (presumably) ∞.

The conclusion that equation 4(b) represents the asymptotic behaviour can now be tested more fully. The quantity

$$Nu_t = Nu - 1{\cdot}44\left(1 - \frac{1708}{Ra}\right)$$

should, according to equation (5), be proportional to $Ra^{1/3}$. To test this, a least square fit of the form $Nu_t = CRa^n$ was taken on all the data (including Goldstein and Chu's) for which $Ra > 70\,000$. The "best-fit-value" of the exponent on Ra was $n = 0{\cdot}3329 \pm 0{\cdot}0074$ (95 per cent confidence limits). The best value of C was $0{\cdot}0580 \pm 0{\cdot}0063$ (95 per cent confidence limits).

Water

It is of interest to make a similar comparison with the available data on water. As mentioned earlier, Fujii and Imura's work with water produces nearly the same value for C_t as determined from Hassan and Mohamed's results for air. Since equation (1) has been found to hold for all Prandtl numbers (except that of mercury), equation (5) might be anticipated to hold for water as well as air. Figure 3 shows the comparison of equation (5) with the combined experimental data of Rossby [6], Goldstein and Chu [3], and Garon and Goldstein [7]. The agreement is good for large and near critical Rayleigh numbers, but is not good at intermediate Rayleigh numbers. It has been found possible to represent the deviation, ΔNu, between equation (5) and the data, by the function

$$\Delta Nu = 2{\cdot}0\left[\frac{Ra^{1/3}}{140}\right]^{1-\ln(Ra^{1/3}/140)}$$

For water then, the Nusselt number dependence on the Rayleigh number is expressed by the equation:

$$Nu = 1 + 1{\cdot}44\left(1 - \frac{1708}{Ra}\right)^{\bullet} + \left[\left(\frac{Ra}{5830}\right)^{1/3} - 1\right] + 2{\cdot}0[Ra^{1/3}/140]^{[1-\ln(Ra^{1/3}/140)]}. \tag{6}$$

Equation (6), plotted in Fig. 3 is seen to be a very close fit to the experimental data over the full Rayleigh number range. (The addition of the ΔNu term to form equation (6) is purely for the purposes of obtaining an empirical fit; however, a physical interpretation is given in a later section.)

The data of Rossby, Goldstein and Chu, and Garon and Goldstein have been chosen for the above fitting of the correlation equation for water since they all show very little scatter and because they compare very favourably with each other in the regions where they overlap. However, it could be argued that only selective data have been for comparison against correlation equation (6) and therefore that the equation lacks universal agreement. A comparison with other (earlier) data for water is shown in Fig. 4. This graph shows all the data for water used in the treatise by O'Toole and Silveston. This includes that of Globe and Dropkin [16], Schmidt and Silveston [26] and Stumpf [27]. The data of Schmidt and Silveston are in very good agreement with equation [6]; those of Stumpf show greater scatter but on average are in quite good agreement; those of Globe and Dropkin "cut through" the present correlation equation, putting them in slight disagreement with other workers.

The additional heat transfer of water over air, characterized by ΔNu, is roughly centred about the Ra number range ($5 \times 10^5 - 5 \times 10^6$) termed "moderate" by Goldstein and Chu [2]. In this range, thermals are most strongly observed in water and one is tempted to ascribe the additional heat transfer to thermals since the existence of thermals was not mentioned in Goldstein and Chu's study of air layers. That thermals should be observed with water and not with air indicates that they are Prandtl number dependent and in fact this would seem reasonable, since the Prandtl number is the ratio of momentum to thermal diffusivities. In a fluid with a high thermal diffusivity (low Prandtl number) packets of fluid leaving the outer edge of the boundary layer will lose heat and take up a temperature equal to the surrounds within a much shorter distance from its starting point than a fluid with a low thermal diffusivity. Presumably thermals leaving the outer side of the boundary layer in air are dissipated

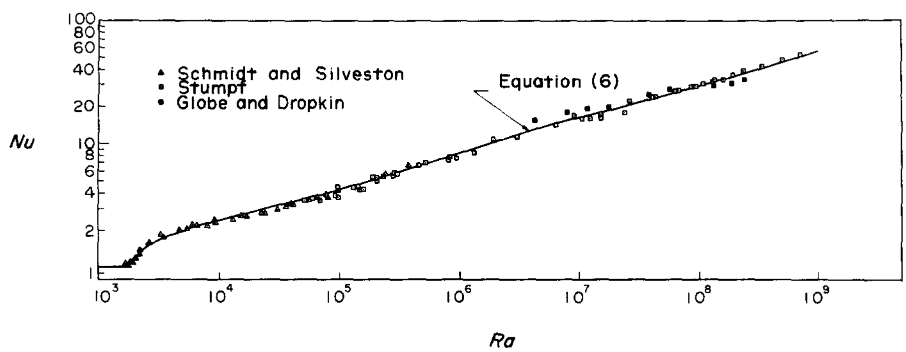

FIG. 4. Comparison of correlation equation (6) with earlier experimental data for water.

Except for the data of Malkus [26] and Krishnamurti [2] which were unavailable, and that of Jakob and Gupta [27] which were demonstrated by O'Toole and Silveston [5] to be inapplicable to the present problem, Figs. 3 and 4 represent a comparison of equation (6) with all published heat-transfer data on water known to the authors.

PHYSICAL INTERPRETATION OF CORRELATION EQUATIONS

It is of interest to attempt to interpret equations (5) and (6) in terms of the observed nature of the flow field at different Rayleigh numbers. On the basis of these equations one might expect a transition at $Ra \simeq 5800$ and in fact a transition is observed for air at $Ra \simeq 4800$ by Krishnamurti [2] and at $6300 < Ra < 10000$ by Willis and Deardorff [25]. In both cases the transition is to time-dependent flow. Since equation (1) represents the heat transfer associated with steady rolls, it is tempting to associate the second term in equation (5) with a correlation of purely spatial temperature and velocity fluctuations, and the third term to a correlation of purely temporal fluctuations. It should be noted, however, that the observed time-dependent transition occurs at a much higher Rayleigh number ($\simeq 30000$ [2,3]) for water.

before they reach the outer side, whereas those in water reach the other side still maintaining some of the temperature field in the opposite boundary layer.

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REFERENCES

1. E. J. Plate, Aerodynamic characteristics of atmospheric boundary layers, A.E.C., Critical Review Series, U.S. Atomic Energy Commission, Div. of Tech. Inf., pp. 99–133 (1971).
2. R. Krishnamurti, On the transition to turbulent convection, Part 2, The transition to time-dependent flow, *J. Fluid Mech.* **42**(2), 309–320 (1970).
3. T. Y. Chu and R. J. Goldstein, Turbulent convection in a horizontal layer of water, *J. Fluid Mech.* **60**(1), 141–159 (1973).
4. R. J. Goldstein and T. Y. Chu, Thermal convection in a horizontal layer of air, *Prog. Heat Mass Transfer* **2**, 55–75 (1969).
5. J. L. O'Toole and P. L. Silveston, Correlations of convective heat transfer in confined horizontal layers, *A.I.Ch.E. Chem. Engng Progr. Symp. Ser.* **57**(32), 81–86 (1961).

6. H. T. Rossby, A study of Benard convection with and without rotation, *J. Fluid Mech.* **36**(2), 309–335 (1969).
7. A. M. Garon and R. J. Goldstein, Velocity and heat transfer measurements in thermal convection, *Physics Fluids* **16**(11), 1818–1825 (1973).
8. G. E. Willis and J. W. Deardorff, Confirmation and renumbering of the discrete heat flux transitions of Malkus, *Physics Fluids* **10**(9), 1861–1866 (1967).
9. R. Krishnamurti, On the transition of turbulent convection, Part 1, The transition from two- to three-dimensional flow, *J. Fluid Mech.* **42**(2), 295–307 (1970).
10. W. V. R. Malkus and G. Veronis, Finite amplitude cellular convection, *J. Fluid Mech.* **4**(1), 225–260 (1958).
11. K. G. T. Hollands, Convectional heat transport between rigid horizontal boundaries after instability, *Physics Fluids* **8**, 389–390 (1965).
12. I. Catton, Natural convection in horizontal liquid layers, *Physics Fluids* **9**, 2521–2522 (1966).
13. W. V. R. Malkus, Outline of a theory of turbulent convection, in *Theory and Fundamental Research in Heat Transfer*, edited by J. A. Clark, pp. 203–212. Pergamon Press, New York (1963).
14. J. R. Herring, Investigation of problems in thermal convection, *J. Atmos. Sci.* **20**, 325 (1963).
15. J. R. Herring, Investigation of problems in thermal convection: rigid boundaries, *J. Atmos. Sci.* **21**, 227 (1964).
16. S. Globe and D. Dropkin, Natural convection heat transfer in liquids confined between two horizontal plates, *J. Heat Transfer* **81C**, 24 (1959).
17. K. G. T. Hollands and L. Konicek, Experimental study of the stability of differentially heated inclined air layers, *Int. J. Heat Mass Transfer* **16**, 1467–1476 (1973).
18. K. G. T. Hollands, Natural convection in horizontal thin-walled honeycomb panels, *J. Heat Transfer* **99**(4), 439–444 (1973).
19. W. Mull and H. Reiher, Der Wärmeschutz von Luftschicten, *Beih. Z. Gesundh.-Ing.* (1), **28**, 1–28 (1930).
20. J. G. A. DeGraaf and E. F. M. Van der Held, The relation between the heat transfer and the convection phenomena in enclosed plane air layers, *Appl. Scient. Res.* **A3**, 393–409 (1953).
21. G. D. Raithby and K. G. T. Hollands, A general method of obtaining approximate solutions to laminar and turbulent free convection problems, *Advances Heat Transfer*. Academic Press, New York (1974), to be published.
22. M. Fishenden and O. A. Saunders, *An Introduction to Heat Transfer*, pp. 92–97. Oxford University Press, Oxford (1950).
23. K. Hassan and S. A. Mohamed, Natural convection from isothermal flat surfaces, *Int. J. Heat Mass Transfer* **13**, 1873–1886 (1970).
24. T. Fujii and H. Imura, Natural convection heat transfer from a plate with arbitrary inclination, *Int. J. Heat Mass Transfer* **15**, 755–767 (1972).
25. G. E. Willis and J. W. Deardorff, Measurements on the development of thermal turbulence in air between horizontal plates, *Physics Fluids* **8**, 2225 (1965).
26. E. Schmidt and P. L. Silveston, *Chem. Engng Progr. Symp. Ser.*, No. 29, **55**, 163 (1959).
27. W. Stumpf, Diplomarbeit, Technische Hochschule Muenchen, Munich, Germany (1957).
28. W. V. R. Malkus, *Proc. Roy. S.* **A225**, 185, 196 (1954).

CORRELATIONS DE TRANSFERT DE CHALEUR EN CONVECTION NATURELLE DANS LES COUCHES HORIZONTALES D'AIR ET D'EAU

Résumé—De nouvelles mesures expérimentales sont présentées sur le transfert de chaleur par convection naturelle à travers une couche horizontale d'air, recouvrant une zone de nombres de Rayleigh variant depuis des valeurs subcritiques jusqu'à $4 . 10^6$. Ces données, une fois jointes à celles de Goldstein et Chu pour l'air, on montre que l'ensemble complet des points expérimentaux présente une dépendance du nombre de Nusselt au nombre de Rayleigh qui est asymptotique à une loi puissance 1/3 lorsque le nombre de Rayleigh tend vers l'infini. Le coefficient de proportionnalité asymptotique est en accord avec celui obtenu à l'aide d'un modèle simple de "couche de conduction" décrit dans l'article. La connaissance de l'asymptote a permis d'obtenir une équation de corrélation simple mais précise, valable dans toute la zone de variation du nombre de Rayleigh. Une équation de corrélation similaire est également obtenue par extension pour l'eau.

KORRELATIONSGLEICHUNGEN FÜR DEN WÄRMEÜBERGANG BEI FREIER KONVEKTION IN HORIZONTALEN LUFT- UND WASSERSCHICHTEN

Zusammenfassung—Es wird über neue Messungen des Wärmetransports bei freier Konvektion in horizontalen Luftschichten berichtet. Der erfaßte Bereich reicht von unterkritischen Rayleigh-Zahlen bis zu $Ra = 4 . 10^6$. Kombiniert mit den Ergebnisse von Goldstein und Chu für Luft zeigt sich, daß die Nusselt-Zahl im gesamten Bereich als Potenzfunktion der Rayleigh-Zahl dargestellt werden kann; der Exponent der Rayleigh-Zahl strebt mit wachsender Rayleigh-Zahl asymptotisch dem Wert 1/3 zu. Der asymptotische Proportionalitätsfaktor stimmt mit dem überein, der aus einem einfachen "Schichtleitmodell", das beschrieben wird, errechnet wurde. Die Kenntnis der Asymptote erlaubte die Aufstellung einer einfachen, jedoch genauen Korrelationsgleichung für den gesamten Bereich der Rayleigh-Zahlen. Mit Hilfe einer Erweiterung konnte eine ähnliche Korrelationsgleichung für Wasser aufgestellt werden.

ОБОБЩАЮЩИЕ УРАВНЕНИЯ ДЛЯ ТЕПЛООБМЕНА ПРИ СВОБОДНОЙ КОНВЕКЦИИ В ГОРИЗОНТАЛЬНЫХ СЛОЯХ ВОЗДУХА И ВОДЫ

Аннотация — Сообщаются новые экспериментальные данные о теплообмене свободной конвекцией через горизонтальный слой воздуха в диапазоне чисел Релея от докритических до 4×10^6. При объединении этих данных с результатами Гольдштейна и Чу для воздуха показано, что полная совокупность экспериментальных точек характеризуется зависимостью числа Нуссельта от числа Релея, показатель степени которой асимптотически стремится к 1/3, когда число Релея приближается к бесконечности. Асимптотический коэффициент пропорциональности согласуется с рассчитанным значением для представленной простой «теплопроводной модели слоя». Знание асимптоты позволило получить простое, но точное обобщающее уравнение, которое справедливо для полного диапазона числа Релея. Подобное обобщающее уравнение получено также для воды.